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Theoretical Recoveries in Filter Cake Reslurrying and Washing

Theoretical equations and graphs are presented for filter cake reslurrying and washing with linear solute sorption.

In the case of washing, the filter cake was represented by a model consisting of a number of mixing cells in series. Due to the complexity of the model, explicit algebraic solutions could not be obtained for countercurrent washing, but numerical data were generated using a computer.

Practical application of the above is outlined.

A. TOMIAK

The Ontario Paper Company, Ltd.
Thorold, Ontario, Canada

SCOPE

The objective of this work was to provide a unified treatment of filter cake reslurrying and washing theory which would permit a direct comparison of recoveries obtainable in the two cases.

The above is relevant because it provides basic mathematical theory and leads to a series of graphs suitable for routine use in chemical engineering practice.

The theory is based on material balances applied to the case of linear solute distribution between solid and liquid phases. The following systems were considered:

1. Filter cake reslurrying, including simple reslurry shown in Figure 1; multiple reslurries, in which the cake is subjected to successive reslurries with fresh wash liquor; and countercurrent reslurries shown in Figure 2.

2. Filter cake washing, including simple washing, in which the cake is permeated with fresh wash liquor, and countercurrent washing, a particular case of which is shown in Figure 3, in which the cake is subjected to successive washes with the washings obtained in the following stages.

In the selected method of presentation, the emphasis was placed on practical aspects of evaluating the recoveries. As a consequence of this, the theoretical loss equations were represented by a series of graphs included in

the paper whereas details of their derivation were put in a supplement. For the general reader and filtration specialist, the graphs describe fully the filter cake reslurrying and washing behavior in a direct way without obscuring the issue with lengthy mathematical theory and being hidden in complicated formulae. Anyone interested in the mathematics of the problem is advised to read the supplement as the complexity of the derivations increases from simple reslurry to countercurrent washing and in the last case is quite formidable.*

CONCLUSIONS AND SIGNIFICANCE

Generalized theoretical loss equations were obtained for the case of linear solute distribution between solid and liquid phases, which are also applicable to the simplest case of no solute sorption. They are summarized in Tables 1 and 2. Using the equations, a series of graphs were prepared from which particular cases may be evaluated.

The main parameter was found to be the effective wash ratio N_E. It was shown that to obtain good recoveries $N_E > 1$ must be used.

Because of limited knowledge of filter cake washing, a theoretical model was developed in which the filter cake is represented by a number of perfect mixing cells in series. Such a model covers the range between perfect mixing within the cake and ideal plug displacement for which experimental data has been reported in the literature. Application of the model to countercurrent washing was not simple and required using a computer to obtain numerical data.

The principal significance of this work is that it provides theoretical background for optimizing the number of reslurry or washing stages and the amount of wash liquor required for most economical operation. It emphasizes also in a quantitative way the well-known advantages of countercurrent treatment which is especially important in view of new equipment developments spurred by our increasing concern about pollution.

It is of interest to note that although the reslurrying and washing operations have been treated separately in the past, they are basically similar. In a wide sense, filter cakes may be considered as mixing devices and the reslurrying and washing operations to be analogous to batch and continuous operation of chemical reactors.

To recover valuable material or to remove undesirable impurities, filter cakes are often subjected to reslurry treatment or direct cake washing.

In both cases the recovery aspects of the operation may be conveniently evaluated from theoretical graphs presented in this paper.

Theoretical recovery equations for various reslurry treatments under conditions of no solute sorption were derived by Hawley (1927) and Baker (1936). The theory is based on applying a single reslurry material balance equation to repeated reslurries. Uniform liquid phase concentrations in each reslurry stage and constant volumetric cake liquid phase hold-up throughout the system are assumed which introduces no mathematical difficulties. The advantages of using countercurrent treatment are clearly shown.

Since direct filter cake washing can also be carried out countercurrently, an analogous problem of obtaining filter cake washing recovery equations under such conditions presents itself. This is a more difficult problem because, in general, transient liquid phase concentrations with respect both to time and position will be obtained in the cake during the course of washing.

At the present time there is no generally acceptable basic washing equation available which could be used as a starting point. This is not surprising due to the known complexity of filtration phenomena resulting in variable, often nonhomogenous and difficult to reproduce filter cake structures. Since even the general theory of miscible fluid displacement in uniform porous beds is not completely developed, as indicated by Scheidegger (1960), it appears too much to expect that a single and generally applicable theoretical filter cake washing equation will be forthcoming.

One may expect that simple cases of filter cake washing behavior will fall in between the extremes of ideal plug flow displacement and complete mixing in the filter cake and should result in fair recoveries, whereas in more complicated cases involving solute sorption, cake cracking, and solute transfer from stagnant films, the recoveries will be poorer. Experimental data given by Choudhury and Dahlstrom (1957) and Purchas (1957) shows that at least some filter cakes exhibit intermediate behavior between plug flow and complete mixing.

It was also found that experimental data frequently conform closely to the so-called "diffusion washing equation," which was proposed by Rhodes (1934) on semi-empirical grounds and actually corresponds to complete mixing within the cake, although it was not so stated and is not always realized. (This becomes evident on expressing the recovery as a fraction of the total amount of solute recoverable with an infinite volume of washings.) The above appears reasonable only for thin cakes which may act as single mixing cells.

It is known in axial dispersion studies that deep porous beds may be considered to consist of a large number of perfectly mixed cells connected in series, as proposed by Kramers and Alberda (1953). Such a model approximates closely the capillary diffusivity equation, which was applied to washing of filter cakes by Butler and Tiedje (1957) and more generally was shown to cover the range from plug flow to perfect mixing by Dobie (1962).

The mixing cell model may be seen to span the range from the Rhodes' equation (one cell) to ideal plug displacement (infinite number of cells) and seems thus to be well suited to represent simple cases of filter cake washing. It not only offers considerable advantages in mathematical handling but results in a model familiar in chemical engineering similar for example to applying theoretical stage concept to extraction in packed towers.

In such a model no close physical interpretation of the mixing cells should be attempted, but rather the filter cake should be pictured as a mixing device equivalent to a certain number of mixing cells which will retain the same mixing efficiency under given physical conditions inde-

* The Supplement has been deposited as Document No. 01996 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 305 East 46 Street, New York 10017 and may be obtained for $2.00 for microfiche or $5.00 for photocopies.

pendent of the concentration of wash liquor applied.

So far as semantics are concerned, it is important to avoid calling the cells stages otherwise they will become confused later with the actual countercurrent washing stages.

THEORETICAL LOSS EQUATIONS

Derivation

Theoretical loss equations were derived by the author under the following assumptions:

1. No solid or liquid losses occur.
2. Perfect separations take place, that is, all suspended solids are retained in the cake which results in clear washings.
3. Volumes of filtrate and wash liquor are additive, that is, total volume of a mixture of filtrate and wash liquor equals the sum of the volumes of filtrate and wash liquor added.
4. The solute is linearly distributed between the solid and liquid phases according to the equation $p = kc$. This gives

$$\frac{S_T}{S_L} = \frac{c\,\Delta V_0 + pZ}{c\,\Delta V_0} = 1 + k\frac{Z}{\Delta V_0} \qquad (1)$$

which is constant in a given system.

5. The volumetric cake liquid phase hold-up is constant throughout, independent of concentration, that is, all cakes contain equal volume of liquid.

This seems reasonable in filter cake reslurries but may require modification for some cases of filter cake washing where cake drying occurs.

The mathematics of filter cake reslurrying and washing are based on material balances. The actual derivations are tedious and are of specialized interest. They are only outlined here and the details are available in a supplement.

The general approach was to manipulate the material balance equations to obtain appropriate expressions for the dimensionless number $\frac{c - c_L}{c_0 - c_L}$ for each system. Since for $p = kc$ the solute loss is

$$L = \frac{c\,\Delta V_0 + pZ}{c_0\,\Delta V_0 + p_0 Z} = \frac{c}{c_0} \qquad (2)$$

we can write

$$\frac{c - c_L}{c_0 - c_L} = \frac{\dfrac{c}{c_0} - \dfrac{c_L}{c_0}}{1 - \dfrac{c_L}{c_0}} = \frac{L - \dfrac{c_L}{c_0}}{1 - \dfrac{c_L}{c_0}} = L^0 \qquad (3)$$

to express the fact that for solute-free wash liquor ($c_L = 0$) the dimensionless number reduces to c/c_0 and is equal to the solute loss.

This permits expressing the final results in the form of theoretical loss equations for solute-free wash liquor and making use of an auxiliary transformation given by Equation (3)

$$L = L^0\left(1 - \frac{c_L}{c_0}\right) + \frac{c_L}{c_0} \qquad (4)$$

to extend the above to cases where the wash liquor is not solute-free.

For example, the material balance equation for a single reslurry shown in Figure 1 is

$$(c - c_L)\,V = (c_0 - c)\,\Delta V_0 + (p_0 - p)Z \qquad (5)$$

which for $p = kc$ may be rearranged to

$$(c - c_L)\,N_v = (c_0 - c)\,\frac{S_T}{S_L} \qquad (6)$$

This gives

$$\frac{c_0 - c}{c - c_L} = \frac{S_L}{S_T}N_V = N_E \qquad (7)$$

from which

$$L_R{}^0 = \frac{c - c_L}{c_0 - c_L} = \frac{1}{N_E + 1} \qquad (8)$$

The theoretical loss equations for multiple reslurries and countercurrent reslurries shown in Figure 2 were obtained by repeated application of the single reslurry equation and are given in Table 1. The theoretical loss equations for

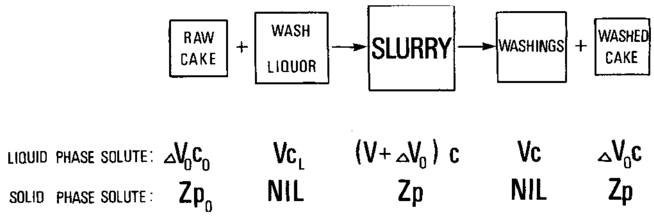

Fig. 1. Single reslurry diagram.

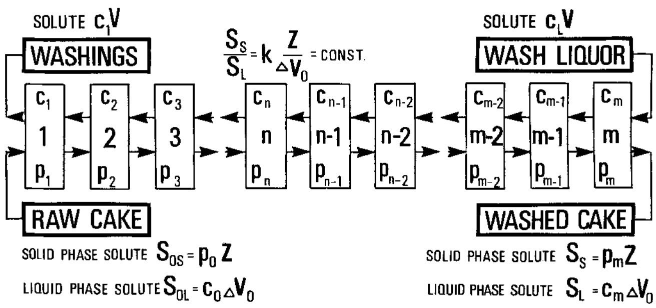

Fig. 2. Countercurrent reslurries diagram.

TABLE 1. THEORETICAL LOSS EQUATIONS FOR RESLURRIES

System	No. of stages	Theoretical loss
Single reslurry	1	$L_R{}^0 = \dfrac{1}{1 + N}$
Multiple reslurries	m	$L^0{}_{MR} = \dfrac{1}{\left(1 + \dfrac{N}{m}\right)^m}$
	∞	$\infty\, L^0{}_{MR} = e^{-N}$
Countercurrent reslurries	m	$L^0{}_{CR} = \dfrac{N - 1}{N^{m+1} - 1} = \dfrac{1}{1 + N + N^2 + N^3 + \ldots + N^m}$
	∞	$\infty\, L_{CR} = 1 - N \quad (N \leq 1)$

filter cake washing are given in Table 2.

For the mixing cells simple cake washing model the starting point was the material balance equation for the ith cell:

$$(c_i - c_{i-1})\, dV = -\left(\frac{\Delta V_0}{j}\, dc_i + \frac{Z}{j}\, dp_i\right) \qquad (9)$$

which was transposed into a differential equation:

$$\frac{dc_i}{dn_E} + c_i = c_{i-1} \qquad (10)$$

Appropriate set of such equations was solved as shown in Table 3. The mathematical treatment was extended to obtain expressions for instantaneous and average concentrations of washings.

The general case of countercurrent washing considered

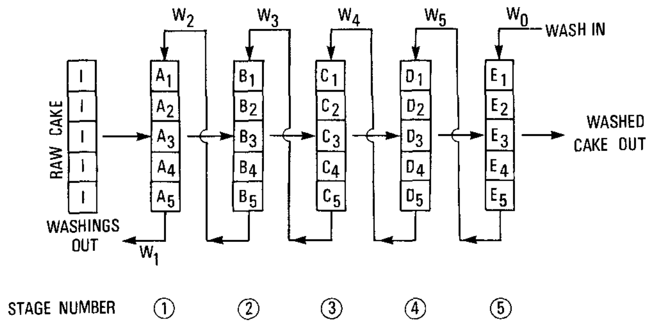

Fig. 3. Countercurrent washing diagram for 5 cells, 5 stages.

in the supplement is too complicated to obtain algebraic solutions in explicit form. A computer program which was prepared and used to obtain numerical solutions is not discussed here but the calculation method is outlined below and applied to a specific case shown in Figure 3.

Because of the complicated nature of the mathematical treatment which involves several equations with various transformations, no attempt was made here to outline the derivations for the case. In essence they involve repeated application of the simple washing equations to the countercurrent mode of operation.

Determination of Theoretical Countercurrent Washing Losses for Mixing Cells Model

In general, the liquor concentrations will be some functions of the wash ratio, which depend on the feed and entering wash liquor concentrations. They may be conveniently expressed by means of the following pairs of parameters

$$\begin{array}{ll}
\alpha = e^{-n} & a = 1 - \alpha \\
\beta = \alpha n & b = a - \beta = 1 - (\alpha + \beta) \\
\gamma = \beta n/2 & c = b - \gamma = 1 - (\alpha + \beta + \gamma) \\
\delta = \gamma n/3 & d = c - \delta = 1 - (\alpha + \beta + \gamma + \delta) \\
\epsilon = \delta n/4 & e = d - \epsilon = 1 - (\alpha + \beta + \gamma + \delta + \epsilon) \\
\vdots & \vdots \qquad\qquad \vdots
\end{array}$$

j pairs of which are needed in the case of j mixing cells.

As an example the system of 5 cells ($j = 5$) and 5

TABLE 2. THEORETICAL LOSS EQUATIONS FOR FILTER CAKE WASHING

System	No. of stages	No. of cells	Theoretical loss
Simple washing	1	1	$L_W{}^0 = e^{-N}$
	1	j	$L_W{}^0 = \frac{1}{j}\left[j + (j-1)\frac{n}{1!} + (j-2)\frac{n^2}{2!} + \ldots + \frac{n^{j-1}}{(j-1)!}\right] e^{-n}$
	1	∞	$\infty L_W{}^0 = 1 - N \quad (N \leq 1)$ $\qquad n = jN$
Countercurrent washing	m	1	$L^0{}_{CW} = \frac{1}{e^N X^{m-1} + Y(1 + X + X^2 + \ldots + X^{m-2})}$ $\left(Y = \frac{e^N - 1}{N} \quad X = e^N - Y\right)$
	∞	1	$\infty L_{CW} = 1 - N \quad (N \leq 1)$
	m	j	See Table 4 or Supplement

TABLE 3. MIXING CELLS MODEL EQUATIONS FOR SINGLE STAGE WASHING

$$c_1 = c_L + (c_0 - c_L)\, e^{-n_E}$$

$$c_2 = c_L + (c_0 - c_L)\,(1 + n_E)\, e^{-n_E}$$

$$c_3 = c_L + (c_0 - c_L)\left(1 + n_E + \frac{n_E{}^2}{2}\right) e^{-n_E}$$

$$\vdots$$

$$c_i = c_L + (c_0 - c_L)\left[1 + \frac{n_E}{1!} + \frac{n_E{}^2}{2!} + \frac{n_E{}^3}{3!} + \ldots + \frac{n_E{}^{i-1}}{(i-1)!}\right] e^{-n_E}$$

$$\frac{c_i - c_L}{c_0 - c_L} = \left[1 + \frac{n_E}{1!} + \frac{n_E{}^2}{2!} + \frac{n_E{}^3}{3!} + \ldots + \frac{n_E{}^{i-1}}{(j-1)!}\right] e^{-n_E} = 1_i{}^0$$

$$L_W{}^0 = \frac{1}{j} \sum_{i=1}^{j} 1_i{}^0 = \frac{1}{j}\left[j + (j-1)\frac{n_E}{1!} + (j-2)\frac{n_E{}^2}{2!} + \ldots + \frac{n_E{}^{j-1}}{(j-1)!}\right] e^{-n_E}$$

stages ($m = 5$) will be considered as shown in Figure 3 where capital letters are used to denote normalized liquor concentrations as follows:

Stage:

1	2	3	4	5

Cell concentration:

$$A_i = \frac{a_i}{a_0} \quad B_i = \frac{b_i}{a_0} \quad C_i = \frac{c_i}{a_0} \quad D_i = \frac{d_i}{a_0} \quad E_i = \frac{e_i}{a_0}$$

Incoming wash:

$$W_2 = \frac{w_2}{a_0} \quad W_3 = \frac{w_3}{a_0} \quad W_4 = \frac{w_4}{a_0} \quad W_5 = \frac{w_5}{a_0} \quad W_0 = \frac{w_0}{a_0}$$

with small letters denoting the actual concentrations (a_0 and w_0 are feed and entering wash liquor concentrations).

All the cell and wash concentrations in the above case may be calculated from the equations shown in Table 4 for any given wash ratio n and an appropriate W_0 as follows:

1. Assume a high trial and error value of $W_2 = W_2^1$
2. Calculate all A's
3. Substitute the A's to obtain W_3
4. Calculate all B's
5. Substitute the B's to obtain W_4
6. Calculate all C's
7. Substitute the C's to obtain W_5
8. Calculate all D's
9. Substitute the D's to obtain W_0^1

The above could be repeated until a correct value of W_0, corresponding to the entering wash liquor, is obtained, but it is more convenient to make use of an auxiliary transformation similar to Equation (4) which permits adjusting the initial guess value of W_2^1 to any desired concentration of entering wash liquor W_0 by means of the formula:

$$W_2 = \frac{W_2^1 - W_0^1}{1 - W_0^1}(1 - W_0) + W_0 \qquad (11)$$

For solute-free wash liquor ($W_0 = 0$) the above reduces to

$$W_2^0 = \frac{W_2^1 - W_0^1}{1 - W_0^1} \qquad (12)$$

Knowing the correct value of W_2 the solute loss in the cake may be found as follows:

1. Repeat steps 1 to 9 starting with W_2
2. Calculate all E's
3. Find the overall loss from

$$L = \frac{\Sigma E}{j} = \frac{E_1 + E_2 + E_3 + E_4 + E_5}{5} \qquad (13)$$

Application of Theoretical Loss Equations

The theoretical loss equations given in Tables 1 and 2 involve as the basic parameter the overall volumetric wash ratio defined by

$$N_V = \frac{\text{Volume of wash liquor}}{\text{Volume of cake liquor hold-up}} \qquad (14)$$

which should be substituted for N in the theoretical loss formulae given in the tables if solute sorption does not occur.

If solute sorption occurs according to $p = kc$, the same formulae are valid but with the N_V replaced by

$$N_E = \frac{S_L}{S_T} N_V \qquad (15)$$

which may be called the effective wash ratio. This is because the starting material balance equations are of the

TABLE 4. THEORETICAL CONCENTRATIONS FOR COUNTERCURRENT WASHING BASED ON MIXING CELLS MODEL ($j = 5, m = 5$)

Cell concentrations	Wash concentrations
$A_1 = \alpha + aW_2$ $A_2 = \alpha + \beta + bW_2$ $A_3 = \alpha + \beta + \gamma + cW_2$ $A_4 = \alpha + \beta + \gamma + \delta + dW_2$ $A_5 = \alpha + \beta + \gamma + \delta + \epsilon + eW_2$	
$B_1 = \alpha A_1 + aW_3$ $B_2 = \alpha A_2 + \beta A_1 + bW_3$ $B_3 = \alpha A_3 + \beta A_2 + \gamma A_1 + cW_3$ $B_4 = \alpha A_4 + \beta A_3 + \gamma A_2 + \delta A_1 + dW_3$ $B_5 = \alpha A_5 + \beta A_4 + \gamma A_3 + \delta A_2 + \epsilon A_1 + eW_3$	$W_3 = \frac{nW_2 - (aA_5 + bA_4 + cA_3 + dA_2 + eA_1)}{n - (a + b + c + d + e)}$
$C_1 = \alpha B_1 + aW_4$ $C_2 = \alpha B_2 + \beta B_1 + bW_4$ $C_3 = \alpha B_3 + \beta B_2 + \gamma B_1 + cW_4$ $C_4 = \alpha B_4 + \beta B_3 + \gamma B_2 + \delta B_1 + dW_4$ $C_5 = \alpha B_5 + \beta B_4 + \gamma B_3 + \delta B_2 + \epsilon B_1 + eW_4$	$W_4 = \frac{nW_3 - (aB_5 + bB_4 + cB_3 + dB_2 + eB_1)}{n - (a + b + c + d + e)}$
$D_1 = \alpha C_1 + aW_5$ $D_2 = \alpha C_2 + \beta C_1 + bW_5$ $D_3 = \alpha C_3 + \beta C_2 + \gamma C_1 + cW_5$ $D_4 = \alpha C_4 + \beta C_3 + \gamma C_2 + \delta C_1 + dW_5$ $D_5 = \alpha C_5 + \beta C_4 + \gamma C_3 + \delta C_2 + \epsilon C_1 + eW_5$	$W_5 = \frac{nW_4 - (aC_5 + bC_4 + cC_3 + dC_2 + eC_1)}{n - (a + b + c + d + e)}$
$E_1 = \alpha D_1 + aW_0$ $E_2 = \alpha D_2 + \beta D_1 + bW_0$ $E_3 = \alpha D_3 + \beta B_2 + \gamma D_1 + cW_0$ $E_4 = \alpha D_4 + \beta D_3 + \gamma D_2 + \delta D_1 + dW_0$ $E_5 = \alpha D_5 + \beta D_4 + \gamma D_3 + \delta D_2 + \epsilon D_1 + eW_0$	$W_0 = \frac{nW_5 - (aD_5 + bD_4 + cD_3 + dD_2 + eD_1)}{n - (a + b + c + d + e)}$

same form after making up the above substitution.

It may be noted that for $N_E < 1$ complete recoveries are not possible even with an infinite number of stages. It is evident that to obtain good recoveries in practice it is necessary to use wash ratios $N_E > 1$. The value of $N_E = 1$ may be called the minimum effective wash ratio. The corresponding volumetric wash ratio is

$$(N_V)\text{min} = \frac{S_T}{S_L}\,(N_E)\text{min} = \frac{S_T}{S_L} = 1 + k\,\frac{Z}{\Delta V_0} \quad (16)$$

The physical interpretation of the above limiting case is that a certain minimum wash volume is required because the concentration of the washings cannot exceed that of the original filtrate. The minimum effective wash ratio corresponds thus to an ideal case in which the concentration of the washings is the same as that of the filtrate in unwashed cake.

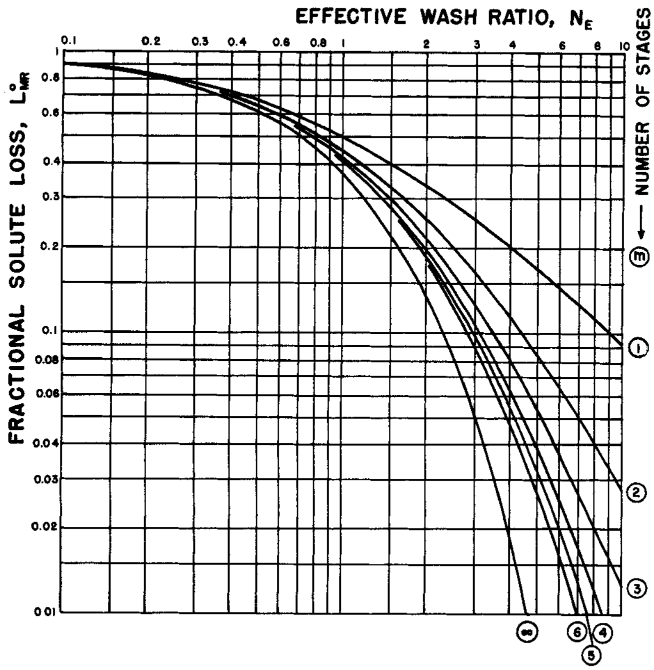

Fig. 4. Theoretical loss curves for multiple reslurries.

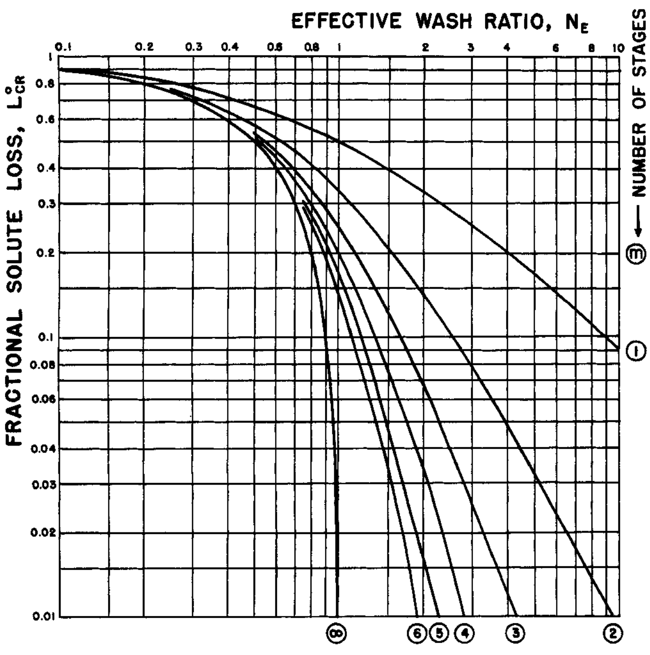

Fig. 5. Theoretical loss curves for countercurrent reslurries.

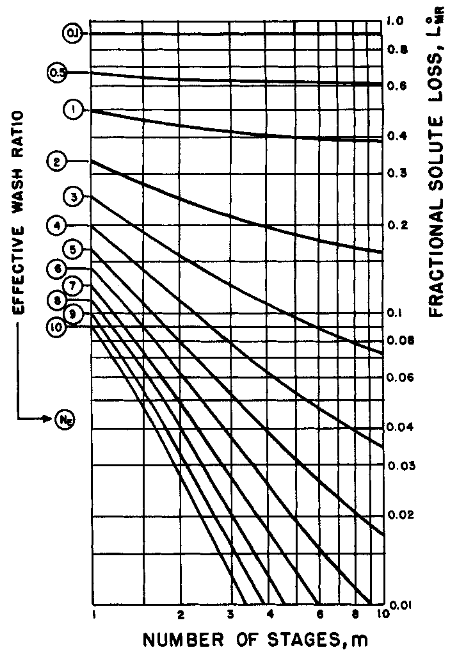

Fig. 6. Theoretical loss curves for multiple reslurries.

It should be realized that any simple filter cake washing theories apply strictly only to ideal cakes whereas real cakes are susceptible to developing local imperfections due to cake cracking, which cause preferential flow along the path of least resistance. To allow for this one may use a simple model in which part of the wash liquor bypasses the cake at infinite speed without any intermixing within the cake. Under such conditions the theoretical loss equations are still valid after putting $N_V = \alpha \dfrac{V}{\Delta V_0}$ where α is the fraction of the wash liquor not bypassing the cake.

GRAPHICAL PRESENTATION

The theoretical loss equations for multiple and countercurrent reslurries are presented graphically in Figures 4 to 9. Figures 6 to 9 are particularly instructive in showing that at low effective wash ratios the recoveries cannot be effectively improved by increasing the number of stages.

The theoretical curves for simple filter cake washing are shown in Figures 10 to 12. Using these curves permits verifying whether the mixing cell model is applicable and finding out how many cells a given cake is equivalent to as a measure of the efficiency of washing.

The theoretical curves for countercurrent filter cake washing represented by the mixing cell model are shown in Figures 13 to 15. They illustrate well the effect of increased efficiency of countercurrent washing causing the narrowing of the spread between complete mixing ($j = 1$) and plug displacement ($j = \infty$).

It is apparent that with noncracking filter cakes countercurrent washing is more efficient than countercurrent reslurrying with the same volume of wash liquor. Although they both approach plug displacement with a large number of stages, the recoveries with single stage washing are better than with single stage reslurry which results in narrowing of the spread of the washing curves between $m = 1$ and $m = \infty$ compared with the reslurry curves of Figure 5.

PRACTICAL APPLICATION OF RESLURRYING AND WASHING THEORY

This is straightforward for reslurries except for attaining equilibrium in cases involving solute sorption. Apart from some variation in filter cake liquor hold-ups which normally should be small, all that is required is that the reslurries be thoroughly mixed.

It is not so simple for filter cake washing because even in the case of no solute sorption and no filter cake cracking, the proposed mixing cell model involves one parameter that cannot be directly measured or predicted reliably: the number of mixing cells. It is a characteristic property of a given cake influenced by the operating conditions (cake and wash liquor distribution, cake dryness, and operating temperature and pressure) which should be experimentally determined for each cake.

This may be conveniently done for aqueous solutions by plotting the experimental data for simple filter cake washing with water and comparing it with the graphs shown in Figures 10 to 12. With negligible solute sorption and filter cake cracking, one may plot the experimental curves directly on the above graphs and estimate the number of cells from their relative position. Although considerable scattering of the experimental data is to be expected, safe estimates of the minimum number of cells the cake is equivalent to may be thus obtained, using which conserva-

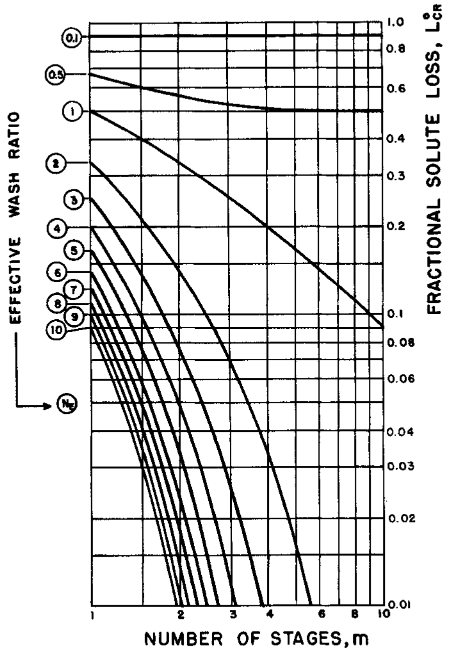

Fig. 7. Theoretical loss curves for counter-current reslurries.

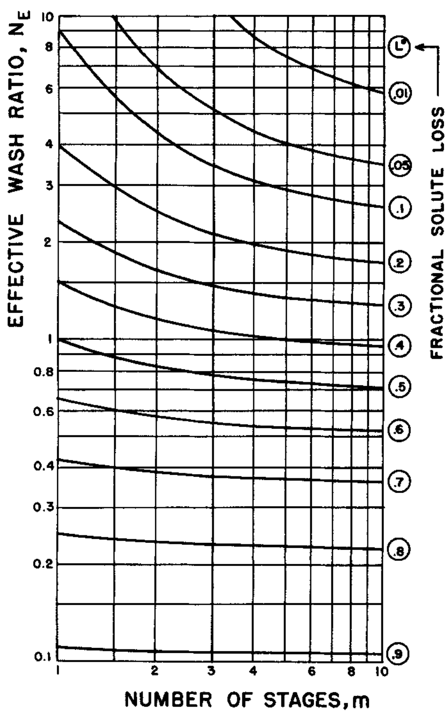

Fig. 8. Theoretical loss curves for multiple reslurries.

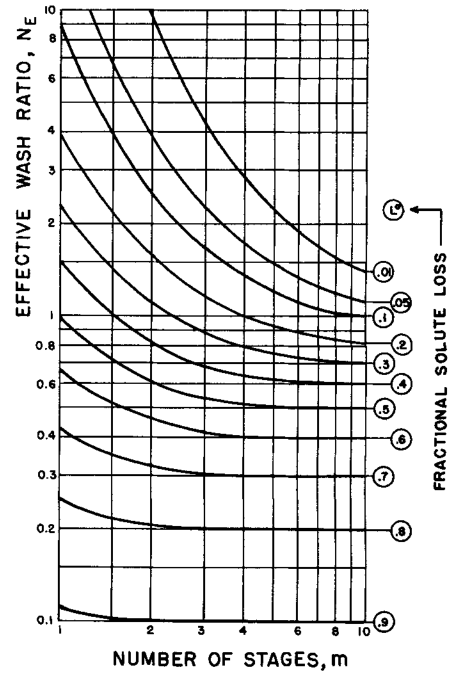

Fig. 9. Theoretical loss curves for counter-current reslurries.

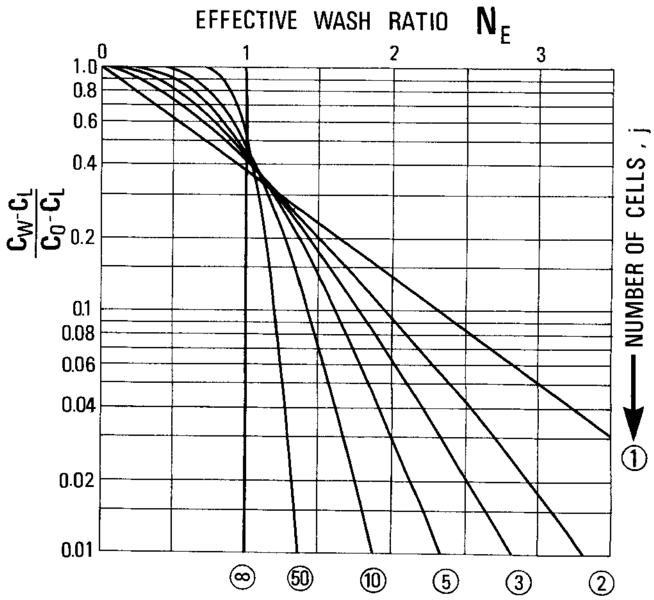

Fig. 10. Theoretical curves for instantaneous concentration of washings for single stage washing based on mixing cells model.

tive predictions may be made for countercurrent washing.

Knowing the number of mixing cells permits running of detailed calculations on a computer for specific numbers of countercurrent stages using a generalized program based on the equations shown in Table 4.

The numerical data thus obtained is useful in estimating the desired number of stages and the wash liquor requirements. It may also be used as a starting point for laboratory simulation work in which synthetic mixes of filtrate

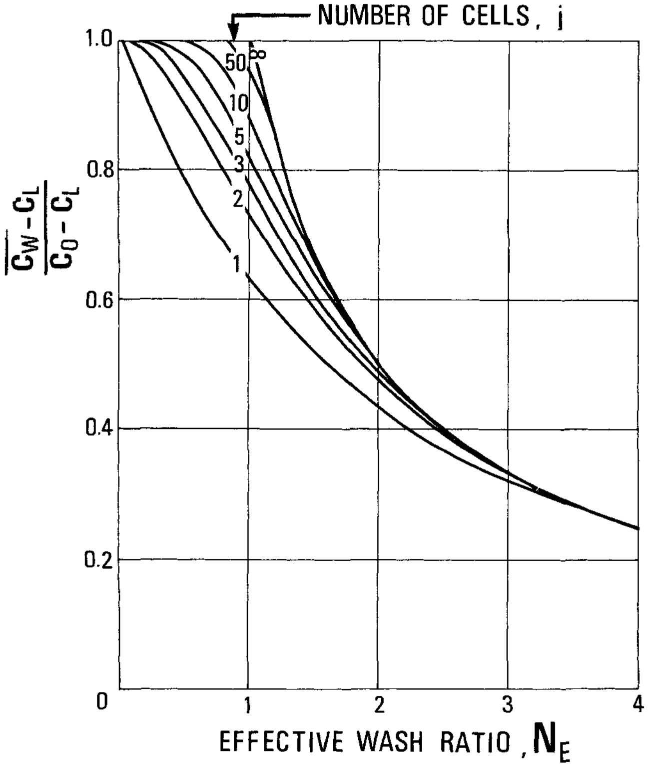

Fig. 11. Theoretical curves for average concentration of washings for single stage washing based on mixing cells model.

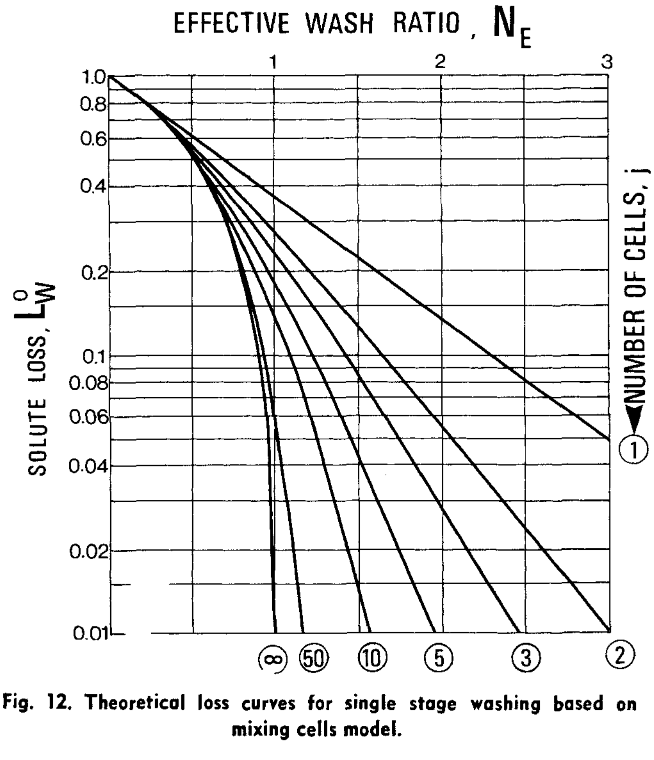

Fig. 12. Theoretical loss curves for single stage washing based on mixing cells model.

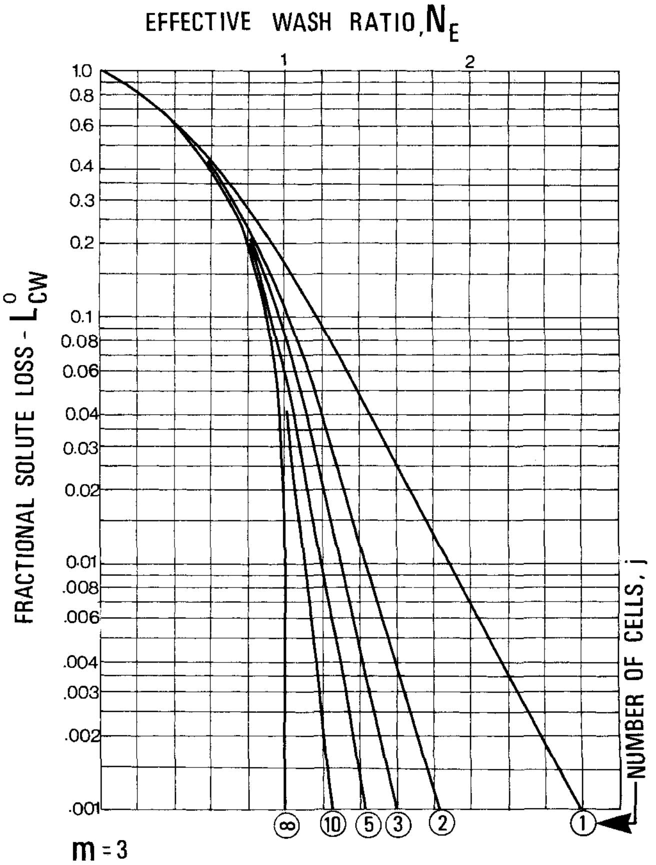

Fig. 13. Theoretical loss curves for 3 stage countercurrent washing based on mixing cells model.

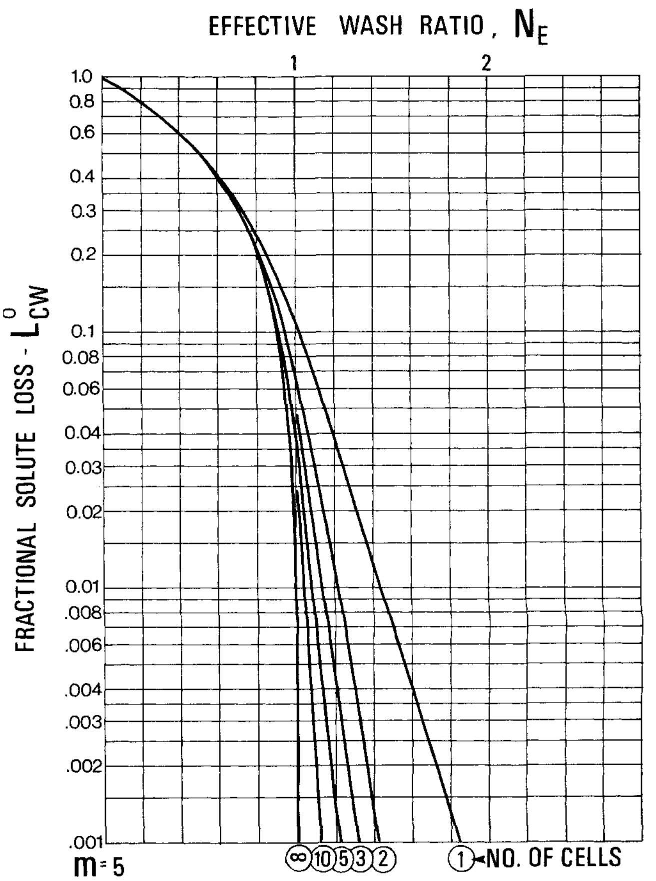

Fig. 14. Theoretical loss curves for 5 stage countercurrent washing based on mixing cells model.

and the wash liquor may be prepared according to computed theoretical concentrations and then applied as successive washes to the cake; if the theory holds true, the washings thus obtained should match the synthetic mixes prepared for the preceding washes as shown in Figure 16 which was obtained in a simulation study of sulfite pulp washing. (The pulp was very easy to wash, and uniform noncracking cakes were obtained.)

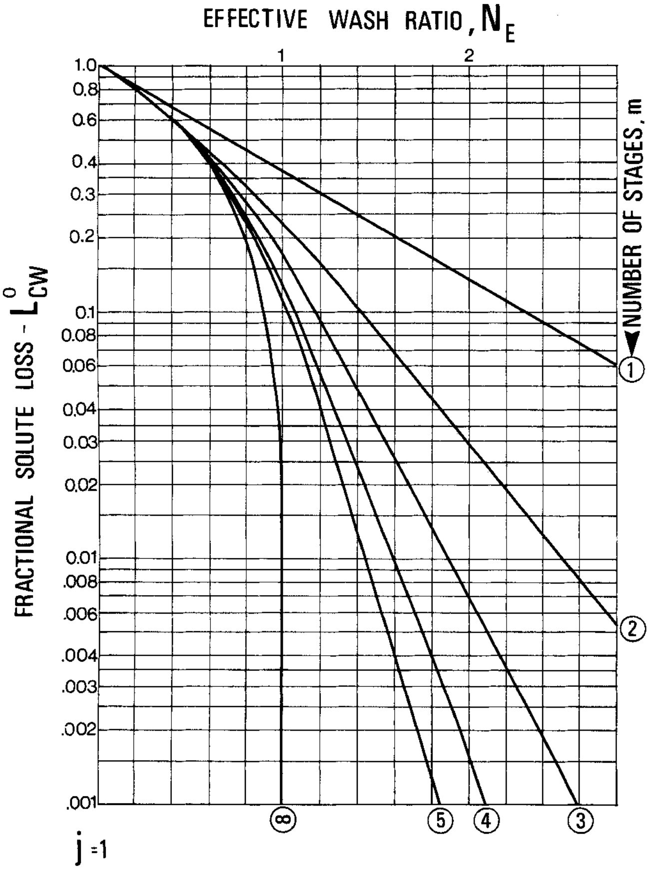

Fig. 15. Theoretical loss curves for countercurrent washing based on single mixing cell model.

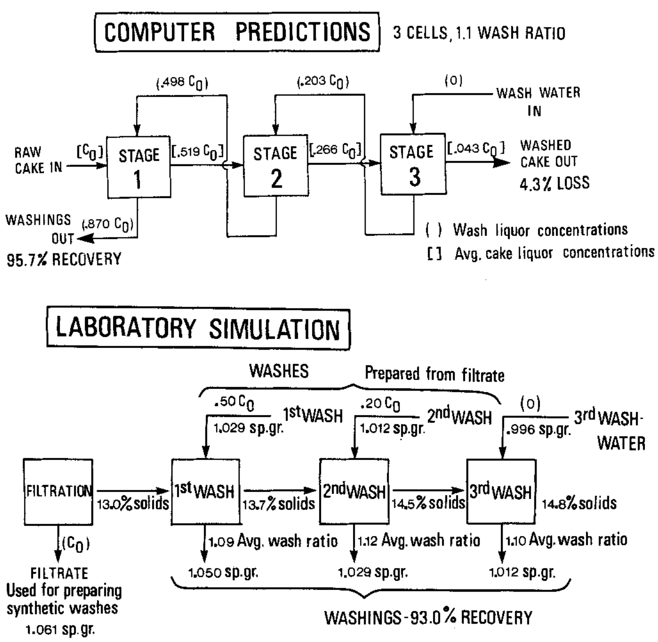

Fig. 16. Comparison between computer's predictions and laboratory simulation.

A word of caution is added here; one should not be carried away by numerical results available from the computer and lose sight of the practical aspects of filter cake washing. The theoretical losses for a few countercurrent stages may be so low as to be meaningless when confronted with nonuniform cake formation and wash liquor distribution problems encountered in practice.

NOTATION

c = solute concentration in liquid phase, g/cm^3
c_0 = initial solute concentration in filter cake liquid phase hold-up, g/cm^3
c_L = solute concentration in entering wash liquor, g/cm^3
c_W = instantaneous solute concentration in effluent washings, g/cm^3
$\overline{c_W}$ = average solute concentration of effluent washings, g/cm^3
e = 2.718.... = base of natural logarithms
j = number of mixing cells, dimensionless
k = distribution coefficient in equation $p = kc$, $\dfrac{\text{g solute/g solids}}{\text{g solute/cm}^3\text{ liquid}}$
L = overall solute loss = fraction of initial total solute content remaining in washed cake, dimensionless
L^0 = overall loss with solute-free wash liquor ($c_L = 0$), dimensionless
m = number of stages, dimensionless
N_V = $\dfrac{V}{\Delta V_0}$ = overall volumetric wash ratio, dimensionless
N_E = $\dfrac{N_V}{1 + k\dfrac{Z}{\Delta V_0}} = \dfrac{S_L}{S_T} N_V$ = overall effective wash ratio, dimensionless
n_V = jN_V = cell volumetric wash ratio, dimensionless
n_E = jN_E = cell effective wash ratio, dimensionless
p = solute concentration in solid phase, g solute/g solids
S_L = weight of solute in cake liquid phase hold-up, g
S_T = total weight of solute in cake, g
V = total volume of washings, cm^3
ΔV_0 = volume of cake liquid phase hold-up, cm^3
Z = weight of solids in cake, g

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